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LUCAS ANTIMAGIC LABELING OF GENERALIZED JAHANGIR GRAPH AND ITSSUBDIVISION GRAPH

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ABSTRACT

A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \to \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).In this paper the Lucas Antimagic Labeling of Generalized Jahangir graph and its Subdivision graph are found.

KEYWORDS: Generalized Jahangir graph, Subdivision of generalized Jahangir graph, Lucas Antimagic graph.

1.INTRODUCTION

In this paper, graph $G(V,E)$ is considered as finite, simple and undirected with p vertices and q edges. A graph labeling is an assignment of integers to the vertices or edges.There are enormous applications for graph labeling in astronomy, theory of coding, circuit design etc. For detailed survey on graph labeling we refer to Gallian[1]. The notion of Antimagic labeling was introduced by N.Hartsfield and G.Ringel in the year 1990. Lucas antimagic labeling is introduced by P.Sumathi and N.Chandravadana[2].

A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \to \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

2.DEFINITIONS

Definition 2.1: Lucas number is defined by

$$L_n = \begin{cases} 2 & \text{if } n = 1 \\ 1 & \text{if } n = 2 \\ L_{n-1} + L_{n-2} & \text{if } n > 2 \end{cases}$$

The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…

Definition 2.2:[2] A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \rightarrow \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \rightarrow \{1,2, \ldots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

Definition 2.3: [3]Generalized Jahangir graph $\mathbf{J(m,n)}$

The generalized Jahangir graph $J(m,n)$is a graph with mn + 1 vertices consisting of a cycle $C_{m,n}$such that one vertex is adjacent to n vertices at a distance m to each other on $C_{m,n}$.

Definition 2.4: [4] $\mathbf{S^1(J(m,n))}$ graph

The graph denoted by $S^1(J(m,n))$ is obtained by inserting an additional vertex into each edge of a cycle $C_{m,n}$ in the graph $J(m,n)$.

Definition 2.5: [4]$\mathbf{S^2(J(m,n))}$ graph

The graph denoted by $S^2(J(m,n))$is obtained from the graph $J(m,n)$ by inserting an additional vertex into each edge which are all incident with the centre vertex $J(m,n)$.

Definition 2.6: [4]Subdivision graph $\mathbf{S(J(m,n))}$

The graph denoted by $S(J(m,n))$ is obtained by inserting an additional vertex into each edge of the graph $J(m,n)$.

3.MAIN RESULTS

Theorem 3.1:

The generalized Jahangir graph $J(m,n)(m \geq 2, n \geq 2)$ is Lucas antimagic graph.

Proof:

Let$V(J(m,n)) = \{u, u_i : 1 \leq i \leq mn\}$

$$E(J(m,n)) = \{u_i u_{i+1} : 1 \leq i \leq mn-1, \quad u_{mn} u_1, \quad uu_{mi} : 1 \leq i \leq n\}$$

Define a function $f: E(J(m,n)) \rightarrow \{L_1, L_2, \ldots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \leq i \leq mn-1$

$f(u_{mn} u_1) = L_{mn}$

$$f(uu_{mi}) = L_{mn+i}, 1 \leq i \leq n$$

The induced function $f^* : V(J(m,n)) \rightarrow \{1,2, \ldots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{mn}$

$$f^*(u_i) = L_{i-1} + L_i, 2 \leq i \leq mn-1, i \not\equiv 0 (mod m)$$

$$f^*(u_{mi}) = L_{mi-1} + L_{mi} + L_{mn+i}, 1 \leq i \leq n$$

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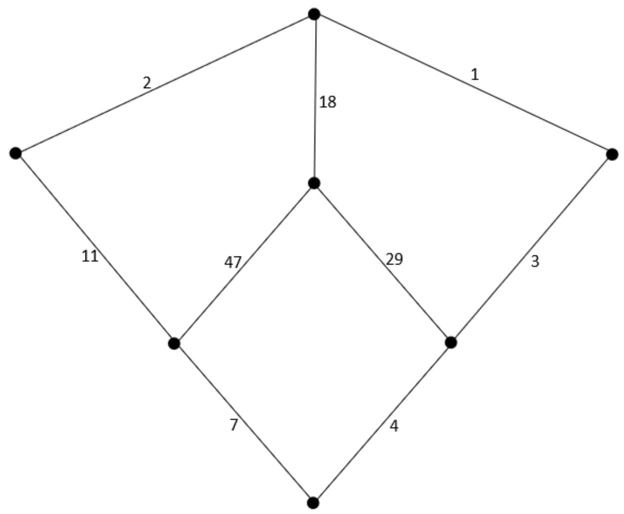
$$f^*(u) = \sum_{i=1}^{n} L_{mn+i}$$

We observe that the vertices are all distinct.

Hence $J(m,n)$ is Lucas antimagic graph.

Example 3.1: The generalized Jahangir graph$J(2,3)$and its Lucas antimagic labeling.



Corollary 3.1.1

The graph $S^1(J(m,n))(m \geq 2, n \geq 2)$ is Lucas antimagic graph.

Proof:

This follows from Theorem 3.1 by replacing the vertices mn+1 by 2mn+1.

$Let\ V(S^1(J(m,n))) = \{u, u_i : 1 \leq i \leq 2mn\}$

$$E(S^1(J(m,n))) = \{u_i u_{i+1} : 1 \leq i \leq 2mn-1, \quad u_{2mn} u_1, \quad uu_{2mi} : 1 \leq i \leq n\}$$

Define a function $f: E(S^1(J(m,n))) \rightarrow \{L_1, L_2, \ldots L_q\}$by

$f(u_i u_{i+1}) = L_i, 1 \leq i \leq 2mn-1$

$f(u_{2mn} u_1) = L_{2mn}$

$$f(uu_{2mi}) = L_{2mn+i}, 1 \leq i \leq n$$

The induced function $f^* : V(S^1(J(m,n))) \rightarrow \{1,2, \ldots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{2mn}$

$$f^*(u_i) = L_{i-1} + L_i, 2 \leq i \leq 2mn-1, i \not\equiv 0 (mod\ 2m)$$

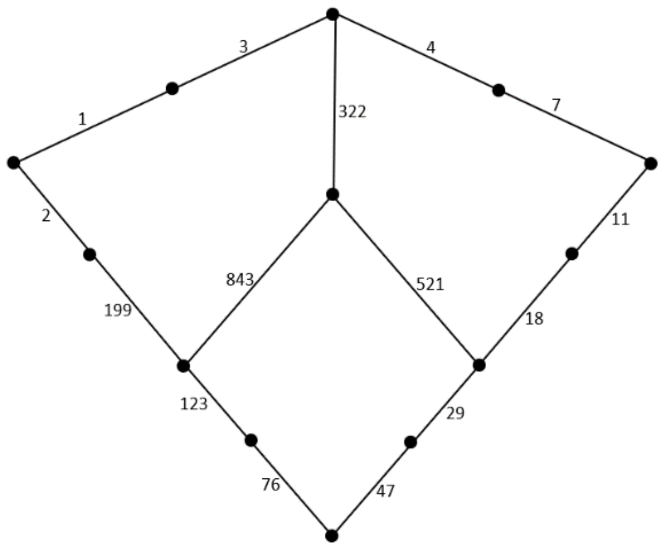
$$f^*(u_{2mi}) = L_{2mi-1} + L_{2mi} + L_{2mn+i}, 1 \leq i \leq n$$

$$f^*(u) = \sum_{i=1}^{n} L_{2mn+i}$$

We observe that the vertices are all distinct.

Hence $S^1(J(m,n))$ is Lucas antimagic graph.

Example 3.1.1: The graph $S^1(J(2,3))$ and its Lucas antimagic labeling.



Theorem 3.2:

The graph $S^2(J(m,n))(m \geq 2, n \geq 2)$ is Lucas antimagic graph.

Proof:

Let $V(S^2(J(m,n))) = \{u, u_i: 1 \leq i \leq mn, v_i: 1 \leq i \leq n\}$

$$E(S^2(J(m,n))) = \{u_i u_{i+1} : 1 \leq i \leq mn-1, \quad u_{mn}u_1, \quad uv_i : 1 \leq i \leq n, v_i u_{mi} : 1 \leq i \leq n\}$$

Define a function $f: E(S^2(J(m,n))) \rightarrow \{L_1, L_2, \dots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \leq i \leq mn-1$

$f(u_{mn}u_1) = L_{mn}$

$f(uv_i) = L_{mn+n+i}, 1 \leq i \leq n$

$$f(v_i u_{mi}) = L_{mn+i}, 1 \leq i \leq n$$

The induced function $f^* : V(S^2(J(m,n))) \rightarrow \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{mn}$

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$$f^*(u_i) = L_{i-1} + L_i, 2 \le i \le mn-1\ , i \not\equiv 0(mod m)$$

$$f^*(u_{mi}) = L_{mi-1} + L_{mi} + L_{mn+i}\ ,1 \le i \le n$$

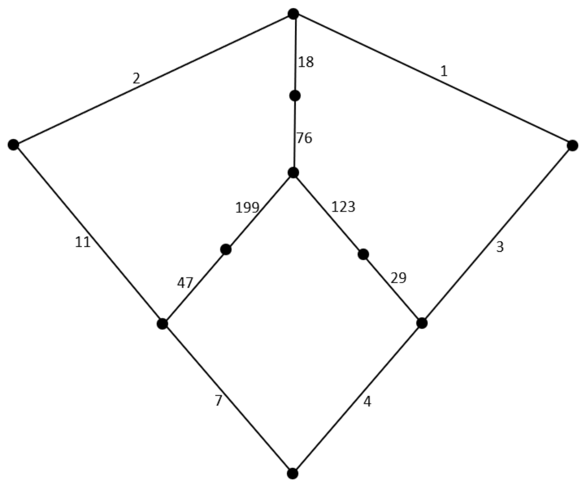
$f^*(v_i) = L_{mn+i} + L_{mn+n+i}\ ,1 \le i \le n$

$f^*(u) = \sum_{i=1}^{n} L_{mn+n+i}$

We observe that the vertices are all distinct.

Hence $S^2(J(m,n))$ is Lucas antimagic graph.

Example 3.2: The graph$S^2(J(2,3))$and its Lucas antimagic labeling.



Corollary 3.2.1:

The subdivision graph $S(J(m,n))(m \ge 2\ , n \ge 2)$ is Lucas antimagic graph.

Proof:

Let $V(S(J(m,n))) = \{u, u_i: 1 \le i \le 2mn\ , v_i: 1 \le i \le n\}$

$$E(S(J(m,n))) = \{u_i u_{i+1} : 1 \le i \le 2mn-1, \quad u_{2mn} u_1\ , \quad uv_i : 1 \le i \le n\ , v_i u_{2mi} : 1 \le \ i \le n\}$$

Define a function $f: E(S(J(m,n))) \to \{L_1, L_2, \dots L_q\}$ by

$f(u_i u_{i+1}) = L_i, 1 \le i \le 2mn-1$

$f(u_{2mn} u_1) = L_{2mn}$

$f(uv_i) = L_{2mn+n+i}\ \ ,1 \le i \le n$

$$f(v_i u_{2mi}) = L_{2mn+i}\ ,1 \le i \le n$$

The induced function $f^* :\ V(S(J(m,n))) \to \{1,2, \dots \sum L_q\}$ is given by

$f^*(u_1) = L_1 + L_{2mn}$

$$f^*(u_i) = L_{i-1} + L_i, 2 \le i \le 2mn - 1, i \not\equiv 0(mod\ 2m)$$

$$f^*(u_{2mi}) = L_{2mi-1} + L_{2mi} + L_{2mn+i}, 1 \le i \le n$$

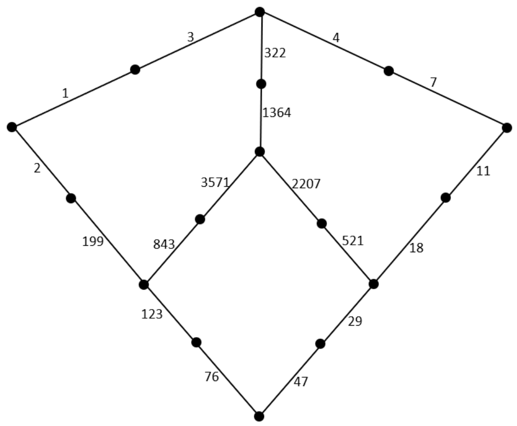
$f^*(v_i) = L_{2mn+i} + L_{2mn+n+i}, 1 \le i \le n$

$f^*(u) = \sum_{i=1}^{n} L_{2mn+n+i}$

We observe that the vertices are all distinct.

Hence $S(J(m,n))$ is Lucas antimagic graph.

Example 3.2.1: The graph$S(J(2,3))$and its Lucas antimagic labeling.



4.CONCLUSION:

In this paper,We have proved thatGeneralized Jahangir Graph and its Subdivision Graph are Lucas antimagic. Similar investigations are in process.

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