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# MODULAR COLORING ON INFLATED GRAPHS

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**Abstract**

A modular sum k-coloring of a graph with vertex set $V(G)$ and he edge set $E(G)$ is an injective map $C: V(G) \to \mathbb{Z}_k, k \geq 2$ where the color sum $\mathcal{S}(v) = \sum_{u \in N(v)} C(v)$ different for adjacent vertices of $G$ that is $\mathcal{S}(u) \neq \mathcal{S}(v)$ for $u, v \in E(G)$. The modular chromatic number $m_c(G)$ of G is the minimum $k$ for which $G$ has a modular sum $k$-coloring. In this paper we established the modular chromatic number of the inflated graph with the name wheel, gear, fan, friendship, and flower graph.

**Keywords:**
Modular $k$-coloring, Modular chromatic number, wheel, gear, fan, friendship, and flower graph.

## 1.Introduction

The graph $G$ with vertex set $V(G)$ and the edge set $E(G)$ is consider in this paper is simple, non-trivial, undirected, and connected.

Let $G$ be an isolated vertex-free graph, the modular coloring is a vertex coloring $C : V(G) \to \mathbb{Z}_k, k \geq 2$ (not a perfect vertex coloring). The coloring is called a modular sum k-coloring for $\mathbb{Z}_k$, if $\mathcal{S}(u) \neq \mathcal{S}(v)$ for $u, v \in E(G)$, let the color sum is defined by $\mathcal{S}(u) = \sum_{w \in N(u)} c(w)$. Let $N(u)$ is the neighborhood of $u$ (set of all adjacent vertices to $u$), for all $u \in G$. The modular chromatic number $m_c(G)$ of $G$ is the minimum $k$ for which $G$ has a modular sum $k$-coloring. [1]

The Inflation or Inflated graph $IG$ of a graph $G$ without isolated vertices is obtained as follows, each vertex $x_i$ of degree $d(x_i)$ of $G$ is replaced by a clique $X_i \cong \mathrm{K}d(x_i)$ and each edge $X_iX_j$ of $G$ is replaced by an edge $uv$ in such a way that $u \in X_i$, $v \in X_j$, and two different edges of $G$ are replaced by non-adjacent edges of $IG$. [6]

In [1] F. Okamoto, E. Salehi, and P. Zhang introduced modular coloring in 2010. In [2,3] N. Paramaguru determined modular colorings of corona product of $C_m$ with $C_n$ and $P_m$ with $C_n$. In [5] P. Sumathi, S. Tamilselvi, found Modular chromatic number of certain cyclic graphs. The objective of this paper is to investigate the modular chromatic number of the inflated graphs of wheel, gear, fan, friendship, and flower graph.

**Definition 1.1 [10]**

The wheel graph $W_n$ is a graph $n \geq 3$ with $n + 1$ vertices, formed by connecting a single vertex to all vertices of an $n$-cycle**.** The wheel graph containing $n + 1$ vertices $\{c, v_1, v_2, \ldots, v_n\}$ and $2n$ edges.

**Definition 1.2 [7]**

A gear graph is obtained from the wheel by adding a vertex between every pair of adjacent vertices of the outer cycle. The gear graph has $2n + 1$ vertices $\{c, v_1, v_2, \ldots, v_{2n}\}$ and $3n$ edges.

**Definition 1.3 [8]**

A fan graph $F_n$ is defined as the graph $K_1 + P_n$ , where $K_1$ is the empty graph on one vertex and $P_n$ , $n \geq 2$ is the path graph on $n$ vertices. A fan graph $F_n$ has $n + 1$ vertices, and $2n - 1$ edges.

**Definition 1.4 [9]**

The friendship graph $T_n$ has $n$ triangles. They all have one common and central vertex. The friendship graph $T_n$ containing $2n+1$ vertices, and 3n edges.

**Definition 1.5 [10]**

A flower graph $Fl_n$ is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm. flower graph $Fl_n$ has $2n+1$ vertices, and $4n$ edges.

## 2. Main results

The inflation or inflated graph is obtained by replacing each vertex $u$ by a clique whose order is equal to the degree of $u$. Let the edge set of I$G$ is a union of the set consisting of edges of the clique along with the edges of $G$. Each vertex in $IG$ belongs to a clique in $IG$. Every vertex of the clique incident with precisely one edge of $G$. [5] .

**Theorem 2.1** Let $n \in N$. Then $|V(IW_n)| = 4n$ and $|E(IW_n)| = nC_2 + 5n$.

**Proof:**

The wheel $W_n$ has exactly $n+1$ vertices, and 2n edges. The order of the inflated graphs of wheel graph is the sum of the degree of the wheel graph $W_n$. That is $|V(IW_n)| = 4n$. Inflated graph of the wheel graph contains a complete graph of order $n$, and also it contains $n$ times of $C_3$ , each $C_3$ join by an edge. Exactly one vertex of each $C_3$ is adjacent to one vertex of complete graph. For size of Inflated graphs of the wheel graph is Sum of $nC_2$ edges from complete graph, $3n$ edges from $n$ times of $C_3$ , $n$ joining edges from complete graph to $C_3$ and $n$ joining edges $\square_3'\square$ will give the total number of edges of $IW_n$. That is $|E(IW_n)| = nC_2 + 3n + n + n = nC_2 + 5n$.

**Theorem 2.2** For any integer $n > 3$, $m_c(IW_n) = n$.

**Proof:**

The graph $IW_n$ is the inflated graph of the wheel graph, $n > 3$ with the vertex set $V(IW_n) = \{u_1, \dots, u_n\} \cup \{v_1, \dots, v_2\} \cup \{w_1, \dots, w_{2n}\}$, and the edge set $E(IW_n) = \left\{x_1, \dots, x_{nC_2}\right\} \cup \{y_1, \dots, y_{3n}\} \cup \{z_1, \dots, z_{2n}\}$. From theorem 2.1 $|V(IW_n)| = 4n$ and $|E(IW_n)| = nC_2 + 5n$. The theorem is proved in two cases.

**Case - 1: When $n$ is even**,

Let $IW_n$ be an inflated graph of wheel graph on $n \geq 3$ it has complete graph of order $n$ as induced sub graph, we know that every complete graph of order $n$ has modular chromatic number $n$, so we need minimum $n$ colors to color the $IW_n$. That is $m_c(IW_n) \geq n$. Let the coloring for the graph is defined by $C: V \rightarrow \{0,1,2\}$ as follows,

$C(u_i) = \{u_i = 0;\ if\ 1 \leq i \leq n\}$,

$C(v_i) = \{v_i = (i-1); if\ 1 \leq i \leq n\}$, and

$$C(w_i) = \begin{cases} 0 \quad i; if\ 1 \leq i \leq 2n, \quad where\ i\ is\ odd\ and\ 4i; if\ i = 1,2,\dots,(\frac{n}{2}-1) \\ 1 \quad\quad 2+4i;\ if\ i = 0,1,2,\dots,(\frac{n}{2}-2) \\ 2 \quad\quad i;\ if\ i = 2n\ and\ 2n-2 \end{cases}$$

Let the modular coloring of inflated graph of wheel graph $IW_n$ is

$\mathcal{S}(u_i) = \{u_i = (i-1)\ ;\ if\ 1 \leq i \leq n\}$,

$$\mathcal{S}(v_i) = \begin{cases} 0 & i;\ 1 \le i \le n, where\ n\ is\ even \\ 1 & i;\ 1 \le i \le n-2, where\ n\ is\ odd\ , \\ 2 & i = n \end{cases}$$

$$\mathcal{S}(w_i) = \begin{cases} 3 & i = 1\ and\ 2n-1 \\ \left(\frac{i}{2}-1\right) & 1 \le i \le 2n, where\ i\ is\ even \\ \left(\frac{i+1}{2}\right) & 3 \le i \le 2n-5,\ where\ i\ is\ odd \\ 1 & i = 2n-3 \end{cases}$$

It gives the modular $n$ - coloring every adjacent vertex receives different color sum. Thus $m_c(IW_n) = n$.

**Case -2: When $n$ is odd,**

Let $IW_n$ be an inflated graph of wheel graph on $n > 3$ it has complete graph of order $n$ as induced sub graph, we know that every complete graph of order $n$ has modular chromatic number $n$. So we need minimum $n$ colors to color the $IW_n$. That is $m_c(IW_n) \ge n$. Let the coloring for the graph is defined by $C: V \to \{0,1,2\}$ as follows,

$C(u_i) = \{u_i = 0;\ if\ 1 \le i \le n\}$, $C(v_i) = \{v_i = (i-1); if\ 1 \le i \le n\}$, and

$$C(w_i) = \begin{cases} 0 & i; 1 \le i \le 2n,\ where\ i\ is\ odd\ and\ 4i; i = 1,2,\dots,\left\lfloor\frac{n}{2}\right\rfloor \\ 1 & 2+4i;\ if\ i = 0,1,2,\dots,(\left\lfloor\frac{n}{2}\right\rfloor - 1) \\ 2 & i = 2n \end{cases}$$

Let the modular coloring of inflated graph of wheel graph $IW_n$ is

$\mathcal{S}(u_i) = \{u_i = (i-1);\ if\ 1 \le i \le n\}$,

$$\mathcal{S}(v_i) = \begin{cases} 0 & i;\ 1 \le i \le n, where\ n\ is\ even \\ 1 & i;\ 1 \le i \le n-2, where\ n\ is\ odd \\ 2 & i = n \end{cases}, \text{ and}$$

$$\mathcal{S}(w_i) = \begin{cases} 3 & i = 1 \\ \left(\frac{i}{2}-1\right) & 1 \le i \le 2n, where\ i\ is\ even \\ \left(\frac{i+1}{2}\right) & 3 \le i \le 2n-3, where\ i\ is\ odd \\ 1 & i = 2n-1 \end{cases}$$

It gives the modular $n$ - coloring every adjacent vertex receives different color sum. Thus $m_c(IW_n)) = n$.

**Theorem 2.3** Let $n \in N$. Then $|V(IG_n)| = 6n$ and $|E(IG_n)| = nC_2 + 7n.$

**Proof:**

Consider the gear graph $G_n$ it has $|V(G_n)| = 2n + 1$ and $|E(G_n)| = 3n$ edges. The order of an inflated graph of a gear graph is sum of the degrees of $G_n$. That is $|V(IG_n)| = 6n$. Inflated graph of a gear graph has a complete graph $K_n$ and $n$ times $C_3$ as induced sub graph, every $C_3$ is join by $P_4$ and also a pendent edge join $K_n$ with each $C_3$. For size of Inflated graphs of the gear graph is Sum of $nC_2$ edges, $n$ times $C_3$ contains $3n$ edges, $n$ times $P_4$ contains $3n$ edges, and also $n$ edges joining $K_n$ with $C_3$'s. That is $|E(IG_n)| = nC_2 + 3n + 3n + n = nC_2 + 7n.$

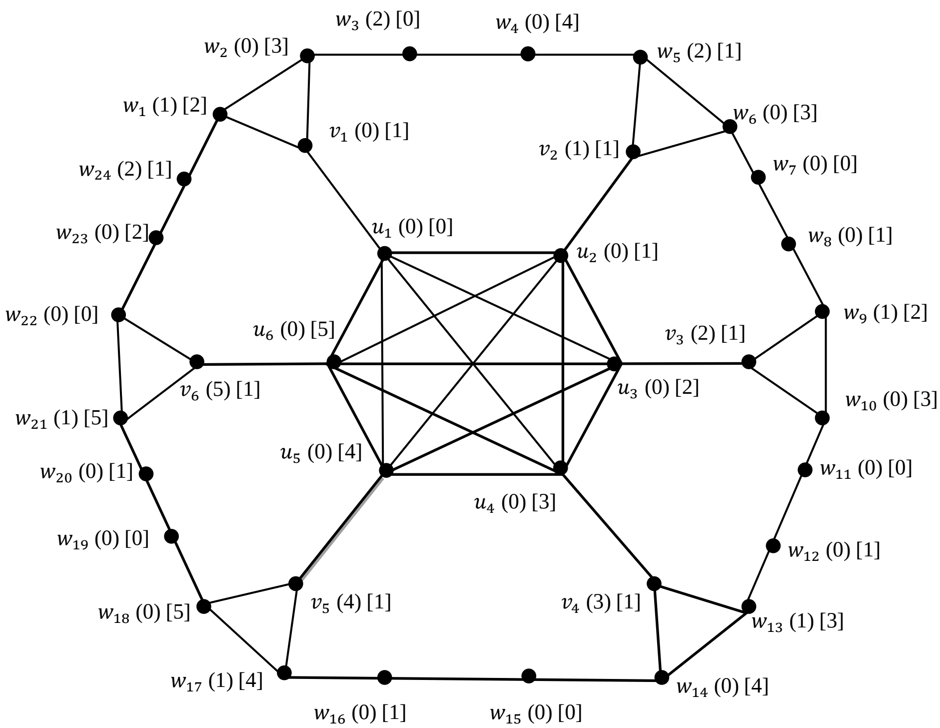


Figure 2.1 [Inflated graph of Gear graph - $IG_6$].

**Theorem 2.4** For any integer $n \geq 3$, $m_c(IG_n) = n$

**Proof:**

The graph $IG_n$, $n \geq 3$ with the vertex set $V(IG_n) = \{u_1, \dots, u_n\} \cup \{v_1, \dots, v_n\} \cup \{w_1, \dots, w_{4n}\}$, and the edge set $E(IG_n) = \{x_1, \dots, x_{nC_2}\} \cup \{y_1, \dots, y_{3n}\} \cup \{z_1, \dots, z_{4n}\}$. Let $|V(IG_n)| = 6n$ and $|E(IG_n)| = nC_2 + 7n$.

Let $IG_n$ be an inflated graph of gear graph, $n \geq 3$ it has complete graph of order $n$ as induced sub graph, we know that every complete graph of order $n$ has modular chromatic number $n$. So we need minimum $n$ colors to color the $IG_n$. That is $m_c(IG_n) \geq n$. Let the coloring for the graph is defined by $C: V \to \{0,1,2, \dots, n-1\}$ as follows,

$C(u_i) = \{u_i = 0;\ if\ 1 \leq i \leq n\}$, $C(v_i) = \{v_i = (i-1); if\ 1 \leq i \leq n\}$, and

$$C(w_i) = \begin{cases} 0 & i; if\ 1 \leq i \leq 4n-1,\ where\ i\ is\ even\ and\ 3+4i; if\ 1 \leq i \leq n-1 \\ 1 & i; if\ i = 1\ and\ 5+4i;\ if\ 1 \leq i \leq n-2 \\ 2 & i = 3,5\ and\ 4n \end{cases}$$

Let the modular coloring of inflated graph of gear graph $IG_n$ is

$\mathcal{S}(u_i) = \{u_i = (i-1);\ if\ 1 \leq i \leq n\}$,

$$\mathcal{S}(v_i) = \begin{cases} 1 & i;\ if\ i = 1\ and\ 3 \leq i \leq n \\ 2 & if\ i = 2 \end{cases}$$ ,and

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$$\mathcal{S}(w_i) = \begin{cases} 0 & 3+4i;\ if\ 0 \le i \le n-2 \\ 1 & 4+4i;\ if\ 1 \le i \le n \\ 2 & n=1\ and\ 4n-1 \\ 3 & n=2 \\ 4 & n=4 \\ i & 1+4i; if\ 1 \le i \le n-1 \\ i+1(mod\ n) & 2+4i; if\ 1 \le i \le n-1 \end{cases}.$$

It gives the modular $n$ - coloring every adjacent vertex receives different color sum, Ref- Fig 2.1. Thus $m_c(IG_n) = n$.

**Theorem 2.5** Let $n \in N$. Then $|V(IF_n)| = 4n-2$ and $|E(IF_n)| = nC_2 + 5n - 5$.

**Proof:**

Consider a fan graph $F_n$ it has $n+1$ vertices and $2n-1$ edges. The order of the Inflated graph of a fan graph is sum of the degrees of $F_n$. That is $|V(IF_n)| = 4n-2$. Inflated graph of a fan graph has a complete graph $K_n$ , $(n-2)$ times $C_3$ , and two times $P_2$ as induced sub graph, $(n-2)C_3$ is join by $P_2$, initial and terminal $C_3$'s are joined with $P_2$ by $P_2$ and also a pendent edge join $K_n$ with $(n-2)C_3$ and two times $P_2$. For size of Inflated graphs of a Fan graph is $nC_2$ edges, $(n-2)$ times $C_3$ contains $3(n-2)$ edges, $(n-3)$ times $P_2$ contains $(n-3)$ edges, and also $n$ edges joining $K_n$ with $(n-2)C_3$ and two times $P_2$. That is $|E(IF_n)| = nC_2 + 3(n-2) + 2 + n + (n-3) + 2 = nC_2 + 5n - 5$.

**Theorem 2.6** For any integer $n \ge 3$, $m_c(IF_n) = n$

**Proof:**

The graph $IF_n$, $n \ge 3$ with the vertex set $V(IF_n) = \{u_1, \dots, u_n\} \cup \{v_1, \dots, v_n\} \cup \{w_1, \dots, w_{2(n-1)}\}$, and the edge set $E(IF_n) = \{x_1, \dots, x_{nC_2}\} \cup \{y_1, \dots, y_n\} \cup \{z_1, \dots, z_{4n-5}\}$. From theorem-2.5, $|V(IF_n)| = 4n-2$ and $|E(IF_n)| = nC_2 + 5n - 5$.

The graph $IF_n$ has a complete graph of order $n$ as an induced sub graph, every complete graph of order $n$ admits modular $n$-coloring. So we need minimum $n$ colors to color the graph $IF_n$. That is $m_c(IF_n) \ge n$.

**Case:1** when $n$ is odd,

Let the coloring for the graph is defined by $C: V \to \{0,1,2 \dots, n-1\}$ as follows,

$C(u_i) = \{u_i = 0;\ if\ 1 \le i \le n\}$, $C(v_i) = \{v_i = (i-1); if\ 1 \le i \le n\}$, and

$$C(w_i) = \begin{cases} 0 & if\ 1 \le i \le 2(n-1),\ where\ i \cong 0,2, and\ 3(mod\ 4) \\ 1 & if\ 1 \le i \le 2(n-1),\ where\ i \cong 1(mod\ 4) \end{cases}$$

Let the modular coloring of inflated graph of fan graph $IF_n$ is

$\mathcal{S}(u_i) = \{u_i = (i-1);\ if\ 1 \le i \le n\}$,

$$\mathcal{S}(v_i) = \begin{cases} 0 & if\ i = n\ and\ 1 \le i \le n, where\ i\ is\ even \\ 1 & if\ 1 \le i \le n-2, where\ i\ is\ odd \end{cases}$$, and

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$$\mathcal{S}(w_i)=\begin{cases}(i-\left\lfloor\frac{i}{2}\right\rfloor) & if\ 1\le i\le 2(n-1)\ ,where\ i\ is\ odd\\ \left(i-\left\lfloor\frac{i}{2}\right\rfloor\right)+1 & if\ 1\le i\le 2(n-2)\ ,where\ i\ is\ even\\ (n-1) & if\ i=2(n-1)\end{cases}$$

**Case: 2** when $n$ is even,

Let the coloring for the graph is defined by $C: V \to \{0,1,2 \dots, n-1\}$ as follows,
$C(u_i)=\{u_i=0;\ if\ 1\le i\le n\}$, $C(v_i)=\{v_i=(i-1); if\ 1\le i\le n\}$, and

$$C(w_i)=\begin{cases}0 & if\ 1\le i\le 2(n-1),\ where\ i\cong 0,2, and\ 3(mod\ 4)\\ 1 & if\ 1\le i\le 2(n-2),\ where\ i\cong 1(mod\ 4)\\ 2 & if\ i=2n-3\end{cases}$$

Let the modular coloring of inflated graph of fan graph $IF_n$ is
$\mathcal{S}(u_i)=\{u_i=(i-1);\ if\ 1\le i\le n\}$,

$$\mathcal{S}(v_i)=\begin{cases}0 & if\ 1\le i\le n\ ,where\ i\ is\ even\\ 1 & if\ 1\le i\le n-2\ ,where\ i\ is\ odd\\ 2 & if\ i=n-1\end{cases}$$, and

$$\mathcal{S}(w_i)=\begin{cases}(i-\left\lfloor\frac{i}{2}\right\rfloor) & if\ 1\le i\le 2n-3\ ,where\ i\ is\ odd\\ \left(i-\left\lfloor\frac{i}{2}\right\rfloor\right)+1 & if\ 1\le i\le 2(n-2)\ ,where\ i\ is\ even\\ 1 & if\ i=2(n-1)\end{cases}$$

It gives the modular $n$ - coloring every adjacent vertex receives different color sum. Thus $m_c(IF_n)=n$.

**Theorem 2.7** Let $n\in N$. Then $|V(IT_n)|=6n$ and $|E(IT_n)|=2nC_2+5n$.

**Proof:**

The friendship graph $T_n$ has $2n+1$ vertices and $3n$ edges. The order of an inflated graph of a friendship graph $T_n$ is sum of the degrees of $T_n$. That is $|V(IT_n)|=6n$. Inflated graph of a friendship graph $IT_n$ has a complete graph $K_{2n}$ , and $n$ times $P_4$ as induced sub graph. We know that $K_{2n}$ contains $2nC_2$ edges, $n$ times $P_4$ contains 3n edges, and also $2n$ edges joining $K_{2n}$ with initial and terminal vertices of $n$ times $P_4$.That is $|E(IT_n)|=2nC_2+3n+2n=2nC_2+5n$.

**Theorem 2.8** For any integer $n\ge 3$, $m_c(IT_n)=2n$

**Proof:**

The graph $IT_n$, $n\ge 3$ with the vertex set $V(IT_n)=\{u_1,\dots,u_{2n}\}\cup\{v_1,\dots,v_{4n}\}$, and the edge set $E(IT_n)=\{x_1,\dots,x_{2nC_2}\}\cup\{y_1,\dots,y_{2n}\}\cup\{z_1,\dots,z_{3n}\}$. From theorem-2.7, $|V(IT_n)|=6n$ and $|E(IT_n)|=2nC_2+5n$.

Let $IT_n$ be an inflated graph of friendship graph on $n\ge 3$, it has complete graph of order $2n$ as induced sub graph, we know that every complete graph of order $n$ has modular chromatic number $n$. so we need minimum $2n$ colors to color the $IT_n$. That is $m_c(IT_n)\ge 2n$.

Let the coloring for the graph is defined by $C: V \to \{0,1,2 \dots, n-1\}$ as follows,

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$C(u_i) = \{u_i = 0;\ if\ 1 \le i \le 2n\}$,

$$C(v_i) = \begin{cases} 0 & if\ i = 1,3+4i; 0 \le i \le n-1, and\ 6+4i; 0 \le i \le n-2 \\ 1 & if\ i = 2 \\ x & x = \left(\frac{j}{2} - 1\right) if j = 4i; 1 \le i \le n \\ y & y = \left(\left\lfloor \frac{j}{2} \right\rfloor\right)\ if\ j = 5+4i; 0 \le i \le n-2 \end{cases}$$ , and

Let the modular coloring of inflated graph of friendship graph $IT_n$ is
$\mathcal{S}(u_i) = \{u_i = (i-1);\ if\ 1 \le i \le n\}$,

$$\mathcal{S}(v_i) = \begin{cases} 0 & if\ i = 2, 4i; 1 \le i \le n, and\ 5+4i; 0 \le i \le n-2 \\ 1 & if\ i = 1 \\ x & x = \left(\frac{j}{2} - 1\right) if j = 6+4i; 0 \le i \le n-2 \\ y & y = \left(\left\lfloor \frac{j}{2} \right\rfloor\right)\ if\ j = 3+4i; 0 \le i \le n-1 \end{cases}$$

It gives the modular $n$ - coloring every adjacent vertex receives different color sum. Thus $m_c(IT_n) = 2n$.

**Theorem 2.9** Let $n \in N$. Then $|V(IFl_n)| = 8n$ and $|E(IFl_n)| = 2nC_2 + 11n$.
**Proof:**

Let $Fl_n$ be a flower graph it has $2n+1$ vertices and $4n$ edges. The order of Inflated graphs of a flower graph is sum of the degrees of $Fl_n$. That is $|V(IFl_n)| = 8n$. Inflated graphs of a flower graph has a complete graph $K_{2n}$ , $(n)$ times $k_4$ , and $nP_2$ as induced sub graph, $nk_4$ is join by n$P_2$. $n$ vertices of $K_{2n}$ joined with n times of $k_4$, remaining $n$ vertices of $K_{2n}$ joined with n times of $P_2$. We know that the complete graph of $K_{2n}$ contains $2nC_2$ edges, $n$ times $k_4$ contains 6n edges, $n$ times $P_2$ contains $n$ edges, also $2n$ edges joining $K_4$ and $P_2$ alternatively with $K_{2n}$, $n$ edges joining each $P_2$with each $K_4$, and also every $K_4$ is joined by an edge that contributes $n$ edges. That is $|E(IFl_n)| = 2nC_2 + 6n + n + 2n + n + n = 2nC_2 + 11n$.

**Theorem 2.10** For any integer $n \ge 3$, $m_c(IFl_n) = 2n$.
**Proof:**

The graph $IFl_n$, $n \ge 3$ with the vertex set $V(IFl_n) = \{p_1, \dots, p_{2n}\} \cup \{q_1, \dots, q_{2n}\} \cup \{r_1, \dots, r_n\} \cup \{s_1, \dots, s_{2n}\} \cup \{t_1, \dots, t_n\}$, and the edge set $E(IFl_n) = \{u_1, \dots, u_{2nC_2}\} \cup \{v_1, \dots, v_{2n}\} \cup \{w_1, \dots, w_{2n}\} \cup \{x_1, \dots, x_{6n}\} \cup \{y_1, \dots, y_n\}$. From theorem- 2.9, $|V(IFl_n)| = 8n$ and $|E(IFl_n)| = 2nC_2 + 11n$.

Let $IFl_n$ be an inflated graph of flower graph on $n \ge 3$ it has complete graph of order $2n$ as induced sub graph, we know that every complete graph of order $n$ has modular chromatic number $n$. So we need minimum $2n$ colors to color the $IFl_n$. That is $m_c(IFl_n) \ge 2n$.

**Case: 1** when $n$ is odd,

Let the coloring for the graph is defined by $C: V \to \{0,1,2 \dots, n-1\}$ as follows,
$C(p_i) = \{p_i = (i-1);\ if\ 1 \le i \le 2n\}$,
$C(q_i) = \{q_i = 0;\ if\ 1 \le i \le 2n\}$,
$C(r_i) = \begin{cases} 0 & if\ 2 \le i \le n \\ 2 & if\ i = 1 \end{cases}$ ,
$C(s_i) = \{s_i = 0;\ if\ 1 \le i \le 2n\}$,

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$$C(t_i) = \begin{cases} n-4 & if\ i = 1\ and\ 2 \le i \le n, where\ i\ is\ even \\ n-2 & 2 \le i \le n, where\ i\ is\ odd \end{cases}.$$

Let the modular coloring of inflated graph of flower graph $IFl_n$ is

$\mathcal{S}(p_i) = \{p_i = (n + (i-1))(mod\ n);\ if\ 1 \le i \le 2n\}$,

$$\mathcal{S}(q_i) = \begin{cases} 1 + 4i(mod\ 2n) & if\ 1 \le i \le 2n\ , where\ i \cong 0,2(mod\ 4) \\ (n-1) + 4i(mod\ 2n) & if\ 1 \le i \le 2n\ , where\ i \cong 1,3(mod\ 4) \end{cases},$$

$\mathcal{S}(r_i) = \{r_i = 0;\ if\ 1 \le i \le n\}$,,

$$\mathcal{S}(s_i) = \begin{cases} 2n-8 & if\ i = 2 \\ 2n-6 & if\ 4 \le i \le 2n, where\ i\ is\ even \\ 2n-2 & if\ i = 1 + 4j,\ 0 \le j \le n-4 \\ 2n-4 & if\ i = 3 + 4j,\ 0 \le j \le n-5 \end{cases}$$

$\mathcal{S}(t_i) = \{t_i = 0;\ if\ 1 \le i \le n\}$.

**Case: 2** when $n$ is even,

Let the coloring for the graph is defined by $C: V \to \{0,1,2 \dots, n-1\}$ as follows,

$C(p_i) = \{p_i = (i-1);\ if\ 1 \le i \le 2n\}$,

$C(q_i) = \{q_i = 0;\ if\ 1 \le i \le 2n\}$,

$$C(r_i) = \begin{cases} 2 & if\ 1 \le i \le n, where\ n\ is\ odd \\ 4 & if\ 1 \le i \le n, where\ n\ is\ even \end{cases},$$

$C(s_i) = \{s_i = 0;\ if\ 1 \le i \le 2n\}$,

$$C(t_i) = \begin{cases} 2n-1 & if\ 1 \le i \le n, where\ i\ is\ odd \\ 2n-3 & if\ 1 \le i \le n, where\ i\ is\ even \end{cases}.$$

Let the modular coloring of inflated graph of flower graph $IFl_n$ is

$\mathcal{S}(p_i) = \{p_i = (n + (i-1))(mod\ 2n);\ if\ 1 \le i \le 2n\}$,

$$\mathcal{S}(q_i) = \begin{cases} 1 & if\ i = 2n \\ n-2 & if\ i = 3 \\ (i-1) & if\ 1 \le i \le 2(n-1), where\ i\ is\ even \\ (i-3) & if\ 5 \le i \le 2n, where\ i \cong 1(mod\ 4) \\ (i-5) & if\ 5 \le i \le 2n, where\ i \cong 3(mod\ 4) \end{cases}$$

$\mathcal{S}(r_i) = \{r_i = 0;\ if\ 1 \le i \le n\}$,

$$\mathcal{S}(s_i) = \begin{cases} 1 & if\ 1 \le i \le 2n, where\ i\ is\ odd \\ 2n-4 & if\ 1 \le i \le n, where\ i\ is\ even \end{cases}$$

$\mathcal{S}(t_i) = \{t_i = 0;\ if\ 1 \le i \le n\}$.

It gives the modular $n$ - coloring every adjacent vertex receives different color sum. Thus $m_c(IFl_n) = 2n$.

## 3. Conclusion:

Coloring is playing essential role in graph theory in that modular coloring is one of the significant area of research. In this paper we have discussed the concept of modular coloring on inflated graphs and derived result for wheel, gear, fan, friendship, and flower graphs. In the future, modular coloring may be applied to a variety of graphs, which can lead to more opportunities for research

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