QUOTIENT-3 CORDIAL LABELING OF SOME JAHANGIR GRAPHS AND ITS SUBDIVISIONS – PART-II

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Abstract

Let G be a graph of order p and size q. Let f: V (G) → $Z_4 - \{0\}$ be a function. For each E (G) define f*: E (G) → Z_3 by f*(uv) = $\left\lceil \frac{f(u)}{f(v)} \right\rceil$ (mod 3) where f(u) ≥ f(v). If the number of vertices having label i and the number of vertices having label j differ by maximum 1, the number of edges having label k and the number of edges having label l differ by maximum 1 then the function f is said to be quotient-3 cordial labeling of G. $1 \leq i, j \leq 3$, $i \neq j$ and $0 \leq k, l \leq 2$, $k \neq l$. In this paper some types Jahangir graphs like $J_{n,3}$, $J_{n,4}$, $J_{3,m}$ and its subdivisions are proved to be quotient-3 cordial.

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Key words: Jahangir, cycle, subdivision, quotient-3 cordial graph

1. Introduction

The graphs considered here are finite, simple, undirected and non trivial. Graph theory has a very fast development in the graph labeling and has a broad range of applications. Refer Gallian [4] for more information. The cordial labeling concept was first introduced by cahit [1]. The quotient-3 cordial labeling has been introduced by P. Sumathi, A. Mahalakshmi and A. Rathi found in [6-9]. They found some family of graphs are quotient-3 cordial. For notations and terminology we follow [3]. If G receives quotient-3 cordial labeling then G is called as quotient-3 cordial graph. The number of vertices having label i denotes $v_f(i)$ and the number of edges having label k denotes $e_f(k)$, $1 \leq i \leq 3$, $0 \leq k \leq 2$. $J_{n,m}$, $m \geq 3$ is a graph containing a cycle C_{mn} along with one more new vertex which is adjacent to m vertices of the cycle at distance n to each other on C_{mn}.

2. Main Result

Definition: Let G be a graph of order p and size q. Let f: V (G) → Z_4 – {0} be a function. For each E (G) define f*: E (G) → Z_3 by f*(uv) = $\left\lceil \frac{f(u)}{f(v)} \right\rceil$ (mod 3) where f (u) ≥ f (v). If the number of vertices having label i and the number of vertices having label j differ by maximum 1, the number of edges having label k and the number of edges having label l differ by maximum 1 then the function f is said to be quotient-3 cordial labeling of G. 1 ≤ i, j ≤ 3, i ≠ j and 0 ≤ k, l ≤ 2, k ≠ l.

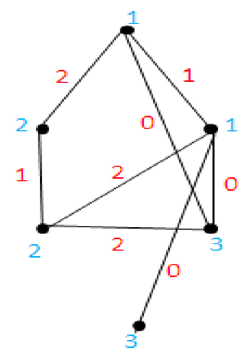

Figure 1 A quotient-3 cordial graph

Theorem: 2.1 The graph $J_{n,3}$,n ≥ 2 quotient-3 cordial.
Proof: Let G be a $J_{n,3}$ graph.
Let V (G) = {u, v_i: 1 ≤ i ≤ 3n}
E (G) = {[(v_iv_{i+1}), (v_1v_{3n}); 1 ≤ i ≤ 3n-1] ∪ [(uv_1), (uv_{n+1}), (uv_{2n+1})]}
Here $|V(G)| = 3n + 1$, $|E(G)| = 3n+3$.
Define f: V (G) → Z_4 - {0}
Labeling of u, v_i, 1 ≤ i ≤ 3n are given below.
Case (i): when n ≡ 0 (mod 6)
f(u) = 2
f(v_1) = 3, f(v_2) = f(v_3) = 2
f(v_{3n}) = 1, f(v_{3n-1}) = 3
For 4 ≤ i ≤ 3n-2
f(v_i) = 1, i ≡ 4, 5 (mod 6)
f(v_i) = 3, i ≡ 0, 3 (mod 6)
f(v_i) = 2, i ≡ 1, 2 (mod 6)
Case (ii): when n ≡ 1 (mod 6)
f(u) = 3
f(v_{3n}) = 1
f(v_{3n-1}) = 3
f(v_{3n-2}) = 2
For 1 ≤ i ≤ 3n-3
f(v_i) = 1, i ≡ 1, 2 (mod 6)
f(v_i) = 3, i ≡ 0, 3 (mod 6)
f(v_i) = 2, i ≡ 4, 5 (mod 6)

Case (iii): when n ≡ 2, 4 (mod 6)
$f(u) = 3$
$f(v_{3n}) = 3$
$f(v_{3n-1}) = f(v_{3n-2}) = 2$
Labeling of v_i, $1 \leq i \leq 3n-3$ are same as in case (ii)
Case (iv): when n ≡ 3, 5 (mod 6)
$f(u) = 2$
$f(v_{3n}) = 3$
$f(v_{3n-1}) = f(v_{3n-2}) = 1$
Labeling of v_i, $1 \leq i \leq 3n-3$ are same as in case (ii)
$e_f(0) = e_f(1) = e_f(2) = n+1$.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0 (mod 6)	n	n+1	n
n ≡ 1, 2, 4 (mod 6)	n	n	n+1
n ≡ 3, 5 (mod 6)	n+1	n	n

Table 1

Theorem: 2.2 A graph G is obtained by subdividing the edges which are all incident with the centre vertex of $J_{n,3}$ is quotient-3 cordial.
Proof: Let G be a graph which is obtained by subdividing the edges which are all adjacent with the centre of vertex of $J_{n,3}$.
Let V (G) = $\{u, v_i, w_j: 1 \leq i \leq 3n, 1 \leq j \leq 3\}$
E (G) = $\{[(v_iv_{i+1}), (v_1v_{3n}); 1 \leq i \leq 3n-1] \cup [(uw_j); 1 \leq j \leq 3] \cup [(w_1v_1), (w_2v_{n+1}), (w_3v_{2n+1})]\}$
Here $|V(G)| = 3n + 4$, $|E(G)| = 3n+6$.
Define f: V (G) → Z_4 - {0}
Labeling of v_i, $1 \leq i \leq 3n$ are given below.
$f(v_i) = 1$, i ≡ 1, 2 (mod 6)
$f(v_i) = 3$, i ≡ 0, 3 (mod 6)
$f(v_i) = 2$, i ≡ 4, 5 (mod 6)
Labeling of u, w_j, $1 \leq j \leq 3$ are given below.
Case (i): when n ≡ 0 (mod 6)
$f(u) = 3$, $f(w_1) = 3$, $f(w_2) = 2$, $f(w_3) = 1$
Case (ii): when n ≡ 1 (mod 6)
$f(u) = 2$, $f(w_1) = 2$, $f(w_2) = 3$, $f(w_3) = 2$
Case (iii): when n ≡ 2 (mod 6)
$f(u) = 2$, $f(w_1) = 3$, $f(w_2) = 1$, $f(w_3) = 2$
Case (iv): when n ≡ 3 (mod 6)
$f(u) = 3$, $f(w_1) = 3$, $f(w_2) = 2$, $f(w_3) = 2$
Case (v): when n ≡ 4 (mod 6)
$f(u) = 2$, $f(w_1) = 3$, $f(w_2) = 2$, $f(w_3) = 1$
Case (vi): when n ≡ 5 (mod 6)
$f(u) = 3$, $f(w_1) = 1$, $f(w_2) = 2$, $f(w_3) = 2$
$e_f(0) = e_f(1) = e_f(2) = n+2$

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0, 3 (mod 6)	n+1	n+1	n+2
n ≡ 1, 2, 4 (mod 6)	n+1	n+2	n+1
n ≡ 5 (mod 6)	n+2	n+1	n+1

Table 2

Theorem: 2.3 The graph $J_{n,4}$ is quotient-3 cordial.
Proof: Let G be a $J_{n,4}$ graph.
Let V (G) = {u, v_i: $1 \leq i \leq 4n$}
E (G) = {[(v_iv_{i+1}), (v_1v_{4n}); $1 \leq i \leq 4n-1$] ∪ [(uv_1), (uv_{n+1}), (uv_{2n+1}), (uv_{3n+1})]}
Here $|V(G)| = 4n + 1$, $|E(G)| = 4n+4$.
Define f: V (G) → Z_4 - {0}
Labeling of u, v_i, $1 \leq i \leq 4n$ are given below.
Case (i): when n ≡ 0 (mod 6)
f(u) = 1
For $1 \leq i \leq 12$,
$f(v_i) = 1$, i ≡ 1, 3 (mod 6)
$f(v_i) = 3$, i ≡ 0, 2 (mod 6)
$f(v_i) = 2$, i ≡ 4, 5 (mod 6)
For $13 \leq i \leq 4n-2$,
$f(v_i) = 1$, i ≡ 1, 2 (mod 6)
$f(v_i) = 3$, i ≡ 0, 3 (mod 6)
$f(v_i) = 2$, i ≡ 4, 5 (mod 6)
$f(v_{4n}) = 2$, $f(v_{4n-1}) = 3$.
Case (ii): when n ≡ 1, 4 (mod 6)
For $1 \leq i \leq 4n$
f(u) = 3
$f(v_i) = 1$, i ≡ 1, 2 (mod 6)
$f(v_i) = 3$, i ≡ 0, 3 (mod 6)
$f(v_i) = 2$, i ≡ 4, 5 (mod 6)
Case (iii): when n ≡ 2 (mod 6)
For $1 \leq i \leq 4n$ ($i \neq 3n+1$)
f(u) = 3
$f(v_i) = 3$, i ≡ 1, 4 (mod 6)
$f(v_i) = 1$, i ≡ 2, 3 (mod 6)
$f(v_i) = 2$, i ≡ 0, 5 (mod 6)
$f(v_{3n+1}) = 2$
Case (iv): when n ≡ 3 (mod 6)
$f(v_{4n}) = 1$, f(u) = 3
$f(v_{3n-1}) = f(v_{3n-2}) = 1$
Labeling of v_i, $1 \leq i \leq 4n-1$ are same as in case (ii)
Case (v): when n ≡ 5 (mod 6)
f(u) = 2, $f(v_{4n}) = 3$, $f(v_{4n-1}) = 1$
Labeling of v_i, $1 \leq i \leq 4n-2$ are same as in case (ii)

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0 (mod 3)	$\frac{4n+3}{3}$	$\frac{4n+3}{3}$ - 1	$\frac{4n+3}{3}$ - 1
n ≡ 1 (mod 3)	$(\frac{4n+2}{3})$	$(\frac{4n+2}{3})$ - 1	$(\frac{4n+2}{3})$
n ≡ 2(mod 3)	$(\frac{4n+1}{3})$	$(\frac{4n+1}{3})$	$(\frac{4n+1}{3})$

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
n ≡ 0 (mod 3)	$\frac{4n+3}{3}$	$\frac{4n+3}{3}$	$\frac{4n+3}{3}+1$
n ≡ 1 (mod 3)	$(\frac{4n+2}{3})$+1	$(\frac{4n+2}{3})$	$(\frac{4n+2}{3})$+1
n ≡ 2(mod 3)	$(\frac{4n+1}{3})$+1	$(\frac{4n+1}{3})$+1	$(\frac{4n+1}{3})$+1

Table 3

Theorem: 2.4 A graph G is obtained by subdividing the edges which are all incident with the centre vertex of $J_{n,4}$ is quotient-3 cordial.

Proof: Let G be a graph which is obtained by subdividing the edges which are all adjacent with the centre of vertex of $J_{n,4}$.

Let V (G) = {u, v_i, w_j: 1 ≤ i ≤ 4n, 1 ≤ j ≤ 4}

E (G) = {[(v_iv_{i+1}), (v_1v_{4n}); 1 ≤ i ≤ 4n-1] ∪ [(uw_j); 1 ≤ j ≤ 4] ∪ [(w_1v_1), (w_2v_{n+1}), (w_3v_{2n+1}), (w_4v_{3n+1})]}

Here $|V(G)|$ = 4n + 5, $|E(G)|$ = 4n+8.

Define f: V (G) → Z_4 - {0} by f(u) = 3

Labeling of v_i, w_j, 1 ≤ i ≤ 4n, 1 ≤ j ≤ 4 are given below.

Case (i): when n ≡ 0 (mod 6)

For 1 ≤ i ≤ 4n

$f(v_i)$ = 1, i ≡ 1, 2 (mod 6)

$f(v_i)$ = 3, i ≡ 0, 3 (mod 6)

$f(v_i)$ = 2, i ≡ 4, 5 (mod 6)

$f(w_1) = f(w_2) = 1$, $f(w_3) = 3$, $f(w_4) = 2$

Case (ii): when n ≡ 1 (mod 6)

For 1 ≤ i ≤ 4n

$f(v_i)$ = 2, i ≡ 1, 2 (mod 6)

$f(v_i)$ = 3, i ≡ 0, 3 (mod 6)

$f(v_i)$ = 1, i ≡ 4, 5 (mod 6)

$f(w_1) =3$, $f(w_2) = 2$, $f(w_3) = f(w_4) = 1$

Case (iii): when n ≡ 2 (mod 6)

Labeling of v_i, 1 ≤ i ≤ 4n are same as in case (i)

Here $f(w_1) = 3$, $f(w_2) = 1$, $f(w_3) = f(w_4) = 2$

Case (iv): when n ≡ 3 (mod 6)

Labeling of v_i, 1 ≤ i ≤ 4n are same as in case (i)

Here $f(w_1) = 3$, $f(w_2) = 2$, $f(w_3) = f(w_4) = 1$

Case (v): when n ≡ 4 (mod 6)

Labeling of v_i, 1 ≤ i ≤ 4n are same as in case (ii)

Here $f(w_1) = 2$, $f(w_2) = f(w_3) = 1$, $f(w_4) = 3$

Case (vi): when n ≡ 5 (mod 6)
Here $f(v_1) = f(v_3) = 1$, $f(v_2) = 3$,
Labeling of v_i, $4 \leq i \leq 4n$ are same as in case (i)
$f(w_1) = 3$, $f(w_2) = f(w_3) = 2$, $f(w_4) = 1$

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n ≡ 0, 3 (mod 6)	$\frac{4n+6}{3}$	$\frac{4n+6}{3}$ - 1	$\frac{4n+6}{3}$
n ≡ 1, 4 (mod 6)	$(\frac{4n+5}{3})$	$(\frac{4n+5}{3})$	$(\frac{4n+5}{3})$
n ≡ 2, 5(mod 6)	$(\frac{4n+7}{3})$	$(\frac{4n+7}{3})$ -1	$(\frac{4n+7}{3})$ -1

Nature of n	$e_f(0)$	$e_f(1)$	$e_f(2)$
n ≡ 0,3 (mod 6)	$\frac{4n+6}{3}$ +1	$\frac{4n+6}{3}$ +1	$\frac{4n+6}{3}$
n ≡ 1,4 (mod 6)	$(\frac{4n+5}{3})$+1	$(\frac{4n+5}{3})$+1	$(\frac{4n+5}{3})$+1
n ≡ 2(mod 6)	$(\frac{4n+7}{3})$	$(\frac{4n+7}{3})$+1	$(\frac{4n+7}{3})$
n ≡ 5(mod 6)	$(\frac{4n+7}{3})$	$(\frac{4n+7}{3})$	$(\frac{4n+7}{3})$+1

Table 4

Theorem: 2.5 The graph $J_{3,m}$, $m \geq 3$ is quotient-3 cordial.
Proof: Let G be a $J_{3,m}$ graph.
Let V (G) = $\{u, v_i: 1 \leq i \leq 3m\}$
E (G) = $\{[(v_1v_{3m}), (v_iv_{i+1}); 1 \leq i \leq 3m-1] \cup [(uv_{3i+1}); 0 \leq i \leq m-1]\}$
Here $|V(G)| = 3m + 1$, $|E(G)| = 4m$.
Define f: V (G) → Z_4 - {0}
When m = 3, f(u) = 2
$f(v_1) = f(v_2) = f(v_7) = f(v_8) = 1$
$f(u_4) = f(u_5) = 2$, $f(u_3) = f(u_6) = f(u_9) = 3$
For $m \geq 4$,
Labeling of v_i, $1 \leq i \leq 3m$ are given below.
For all m, f(u) = 3.
Case (i): when m ≡ 0 (mod 6)
Subcase(i): For $1 \leq i \leq 2m+3$
$f(v_i) = 1$, i ≡ 1, 2 (mod 6)
$f(v_i) = 3$, i ≡ 0, 3 (mod 6)
$f(v_i) = 2$, i ≡ 4, 5 (mod 6)
Subcase(i): For $2m+4 \leq i \leq 3m$
$f(v_i) = 3$, i ≡ 1, 4 (mod 6)
$f(v_i) = 2$, i ≡ 0, 5 (mod 6)
$f(v_i) = 1$, i ≡ 2, 3 (mod 6)

Case (ii): when m ≡ 1 (mod 6)
Subcase(i): For $1 \leq i \leq 2m+4$
$f(v_i) = 2$, $i \equiv 1, 2 \pmod 6$
$f(v_i) = 3$, $i \equiv 0, 3 \pmod 6$
$f(v_i) = 1$, $i \equiv 4, 5 \pmod 6$
Subcase(i): For $2m+5 \leq i \leq 3m-3$
$f(v_i) = 3$, $i \equiv 1, 4 \pmod 6$
$f(v_i) = 2$, $i \equiv 2, 3 \pmod 6$
$f(v_i) = 1$, $i \equiv 0, 5 \pmod 6$
For $3m-2 \leq i \leq 3m$
$f(v_{3m}) = 2$, $f(v_{3m-1}) = 1$, $f(v_{3m-2}) = 3$
Case (iii): when m ≡ 2 (mod 6)
Labeling of v_i, $1 \leq i \leq 2m+5$ are same as in subcase (i) of case(i)
Labeling of v_i, $2m+6 \leq i \leq 3m$ are same as in subcase (ii) of case(i)
Case (iv): when m ≡ 3 (mod 6)
Labeling of v_i, $1 \leq i \leq 2m+3$ are same as in subcase (i) of case(ii)
For $2m+4 \leq i \leq 3m-3$
$f(v_i) = 3$, $i \equiv 1, 4 \pmod 6$
$f(v_i) = 1$, $i \equiv 0, 5 \pmod 6$
$f(v_i) = 2$, $i \equiv 2, 3 \pmod 6$
Labeling of v_i, $3m-2 \leq i \leq 3m$ are same as in case(ii)
For m = 3, $v_f(1) = m+1$, $v_f(2) = v_f(3)$
For m > 3, $v_f(1) = v_f(2) = m$, $v_f(3) = m+1$

Nature of m	$e_f(0)$	$e_f(1)$	$e_f(2)$
m ≡ 0,3 (mod 6)	$\frac{4m}{3}$	$\frac{4m}{3}$	$\frac{4m}{3}$
m ≡ 1 (mod 6)	$(\frac{4m-1}{3})+1$	$(\frac{4m-1}{3})$	$(\frac{4m-1}{3})$
m ≡ 2(mod 6)	$(\frac{4m+1}{3})$	$(\frac{4m+1}{3})$	$(\frac{4m+1}{3})$ - 1
m ≡ 4(mod 6)	$(\frac{4m-1}{3})$	$(\frac{4m-1}{3})$ +1	$(\frac{4m-1}{3})$
m ≡ 5(mod 6)	$(\frac{4m+1}{3})$	$(\frac{4m+1}{3})$	$(\frac{4m+1}{3})$ -1

Table 5

Theorem: 2.6 A graph G is obtained by subdividing the edges which are incident with the centre vertex of $J_{3,m}$, m = 6, 12, 18, … are quotient-3 cordial.
Proof: Let G be a graph which is obtained by subdividing the edges which are adjacent with the centre vertex of $\mathbf{J_{3,m}}$.
Let V (G) = $\{u, v_i, w_j: 1 \leq i \leq 3m, 1 \leq j \leq m\}$
E (G) = $\{[(v_1v_{3m}), (v_iv_{i+1}); 1 \leq i \leq 3m-1] \cup [(uw_j); 1 \leq j \leq m] \cup [w_{j+1}v_{3j+1}); 0 \leq j \leq m-1]\}$
Here $|V(G)| = 4m + 1$, $|E(G)| = 5m$.
Define f: V (G) → Z_4 - {0} by f(u) = 3
Labeling of v_i, w_j; $1 \leq i \leq 3m$, $1 \leq j \leq m$ are given below.
For $1 \leq i \leq 3m$
$f(v_i) = 3$, $i \equiv 1, 4 \pmod 6$
$f(v_i) = 1$, $i \equiv 2, 3 \pmod 6$

$f(v_i) = 2$, $i \equiv 0, 5 \pmod 6$
For $1 \leq j \leq m$
$f(w_j) = 1$, $j \equiv 1 \pmod 3$
$f(w_j) = 3$, $j \equiv 2 \pmod 3$
$f(w_j) = 2$, $j \equiv 0 \pmod 3$
Here $v_f(1) = v_f(2) = \frac{4m}{3}$, $v_f(3) = \frac{4m}{3} + 1$
$e_f(0) = e_f(1) = e_f(2) = \frac{5m}{3}$

Theorem: 2.7 A graph G is obtained by subdividing the edges which are not incident with the centre vertex of $J_{3,m}$ is quotient-3 cordial.
Proof: Let G be a graph which is obtained by subdividing the edges which are not incident with the centre vertex of $J_{3,m}$.
Let $V(G) = \{u, v_i : 1 \leq i \leq 6m\}$
$E(G) = \{[(v_1v_{6m}), (v_iv_{i+1}); 1 \leq i \leq 6m-1] \cup [(uv_{6i+1}); 0 \leq i \leq m-1]\}$
Here $|V(G)| = 6m + 1$, $|E(G)| = 7m$.
Define $f: V(G) \to Z_4 - \{0\}$ by $f(u) = 3$
Labeling of v_i ; $1 \leq i \leq 6m$ are given below.
For $1 \leq i \leq 3m$
Case (i): when $m \equiv 0, 3 \pmod 6$
For $1 \leq i \leq 6m$
$f(v_i) = 1$, $i \equiv 1, 3, 10, 11, 14, 15 \pmod{18}$
$f(v_i) = 3$, $i \equiv 2, 6, 9, 12, 13, 16 \pmod{18}$
$f(v_i) = 2$, $i \equiv 4, 5, 7, 8, 17, 18 \pmod{18}$
Case (ii): when $m \equiv 1 \pmod 6$
Subcase(i): Labeling of v_i, $1 \leq i \leq 6(m-4)$ are same as in case (i)
Subcase(ii): For $6(m-4)+1 \leq i \leq 6m$,
$f(v_i) = 1$, $i \equiv 1, 2, 10, 11, 14, 15 \pmod{18}$
$f(v_i) = 3$, $i \equiv 3, 6, 9, 12, 13, 16 \pmod{18}$
$f(v_i) = 2$, $i \equiv 4, 5, 7, 8, 17, 18 \pmod{18}$
Case (iii) when $m \equiv 2 \pmod 6$
Labeling of v_i, $1 \leq i \leq 6(m-5)$ are same as in case (i)
Labeling of v_i, $6(m-5) + 1 \leq i \leq 6m$ are same as in subcase(ii)of case (ii)
Case (iv) when $m \equiv 4 \pmod 6$
Labeling of v_i, $1 \leq i \leq 6(m-4)$ are same as in case (i)
Labeling of v_i, $6(m-4) + 1 \leq i \leq 6m$ are same as in subcase(ii)of case (ii)
Case (iii) when $m \equiv 5 \pmod 6$
Labeling of v_i, $1 \leq i \leq 6(m-5)$ are same as in case (i)
Labeling of v_i, $6(m-5) + 1 \leq i \leq 6m$ are same as in subcase(ii)of case (ii)
For all m, $v_f(1) = v_f(2) = 2m$, $v_f(3) = 2m+1$

Nature of m	$e_f(0)$	$e_f(1)$	$e_f(2)$
$m \equiv 0, 3 \pmod 6$	$\frac{2mn+m}{3}$	$\frac{2mn+m}{3}$	$\frac{2mn+m}{3}$
$m \equiv 1, 4 \pmod 6$	$\frac{2mn+m-1}{3}$	$\frac{2mn+m-1}{3}+1$	$\frac{2mn+m-1}{3}$
$m \equiv 2, 5 \pmod 6$	$\frac{2mn+m+1}{3} - 1$	$\frac{2mn+m+1}{3}$	$\frac{2mn+m+1}{3}$

Table 6

Theorem: 2.8 The graph $S(J_{3,m})$ is quotient-3 cordial.
Proof: Let G be a $S(J_{3,m})$ graph
Let V (G) = $\{u, v_i, w_j : 1 \leq i \leq 6m, 1 \leq j \leq m\}$
E (G) = $\{[(v_1v_{6m}), (v_iv_{i+1}); 1 \leq i \leq 6m-1] \cup [(uw_j); 1 \leq i \leq m] \cup [(w_{j+1}v_{6j+1}); 0 \leq i \leq m-1]\}$
Here $|V(G)| = 7m + 1$, $|E(G)| = 8m$.
Define f: V (G) → Z_4 - {0} by f(u) = 3
Labeling of v_i, w_j ; $1 \leq i \leq 6m$, $1 \leq j \leq m$ are given below.
For $1 \leq i \leq 6m$
$f(v_i) = 1$, $i \equiv 1, 2 \pmod 6$
$f(v_i) = 3$, $i \equiv 0, 3 \pmod 6$
$f(v_i) = 2$, $i \equiv 4, 5 \pmod 6$
For $1 \leq j \leq m$
$f(w_j) = 1$, $j \equiv 1 \pmod 3$
$f(w_j) = 2$, $j \equiv 2 \pmod 3$
$f(w_j) = 3$, $j \equiv 0 \pmod 3$

Nature of m	$v_f(1)$	$v_f(2)$	$v_f(3)$
$m \equiv 0 \pmod 3$	$\frac{mn+4m}{3}$	$\frac{mn+4m}{3}$	$\frac{mn+4m}{3}+1$
$m \equiv 1 \pmod 3$	$\frac{mn+4m+2}{3}$	$\frac{mn+4m+2}{3} - 1$	$\frac{mn+4m+2}{3}$
$m \equiv 2 \pmod 3$	$\frac{mn+4m+1}{3}$	$\frac{mn+4m+1}{3}$	$\frac{mn+4m+1}{3}$

Nature of m	$e_f(0)$	$e_f(1)$	$e_f(2)$
$m \equiv 0 \pmod 3$	$\frac{mn+5m}{3}$	$\frac{mn+5m}{3}$	$\frac{mn+5m}{3}$
$m \equiv 1 \pmod 3$	$\frac{mn+5m+1}{3}$	$\frac{mn+5m+1}{3}$	$\frac{mn+5m+1}{3} - 1$
$m \equiv 2 \pmod 3$	$\frac{mn+5m-1}{3}$	$\frac{mn+5m-1}{3}$	$\frac{mn+5m-1}{3}+1$

Table 7

Conclusion

In this paper, some Jahangir graphs and its subdivisions graphs have been proved to be quotient-3 cordial. The quotient-3 cordial labeling of some more different kinds of graphs and graph families shall be explored in future.

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