MOD(k) VERTEX MAGIC LABELING OF TREES WITH DIAMETER LESS THAN FOUR

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Abstract: Let G be a simple, undirected and non trivial graph containing p vertices and q edges. For any integer k≥2,$l \in Z_k$ and there exists an injective function f:V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+ l, $\left[\frac{k}{2}\right]$+ l +1,$\left[\frac{k}{2}\right] + k$, $\left[\frac{k}{2}\right]$+k+l, $\left[\frac{k}{2}\right]$+k+ l +1,..., $\left[\frac{k}{2}\right]$+k(p-1)} such that for every edge (e=uv)ϵE(G), the mapping f^*:E(G) → Z_k defined by f^*(uv)=(f(u)+f(v))(mod k)= l is a constant mapping. The function f is said to be a Mod(k) vertex magic labeling of G. A graph G is called Mod(k) vertex magic graph if it admits a Mod(k) vertex magic labeling. In this paper, we prove that trees with diameter less than four are Mod(k) vertex magic graphs.

Keywords: Diameter, Graph Labeling, Mod(k) Vertex Magic Labeling, Tree.

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Introduction: In this paper, we deal with only simple, connected and non-trivial graph G=(V(G),E(G)) among vertex set V(G) and edge set E(G). Throughout this paper | V(G) | =p and | E(G) |=q respectively.

The concept of graph labeling has accomplished a lot of popularity in the area of graph theory. This graph labeling are very useful in Mathematical models for a wide range of applications being X-ray, Crystallography, Coding theory, Radar, Cryptography, Communication networks design, Astronomy, Circuit design and, Database Management.

A **tree** is a connected acyclic graph. If G(V,E) is a connected graph, the distance d(u,v) between two vertices u and v of G is defined as the minimum length of a u-v path in G. The eccentricity e(v) of a vertex v is max d(u,v) where e(v) is the distance between v and a vertex farthest from v. The **diameter** of G is the maximum eccentricity [1].

Throughout this work P_n denotes the **Path** on n-vertices. The **Bistar ($B_{m,n}$)** is the graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2. The edge of K_2 is called the central edge of $B_{m,n}$ and the vertices of K_2 are called the central vertices of $B_{m,n}$ [1].

In 2016, P.Sumathi and B.Fathima [4,5,6,7,8] defined Mod(k) vertex magic labeling and Mod(k) vertex magic graph. Let G be a simple, undirected and non trivial graph with p vertices and q edges. For any integer k≥2,$l \in Z_k$ and there exists an one- to -one mapping f:V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+l, $\left[\frac{k}{2}\right]$+l+1 ,..., $\left[\frac{k}{2}\right]$+k(p-1)} such that for every edge (e=uv)ϵE(G), the mapping f^*:E(G) → Z_k defined by f^*(e=uv)=(f(u)+f(v))(mod k)= l is a constant mapping. The function f is called a Mod(k) vertex magic labeling of G. A graph G is called Mod(k) vertex magic graph if it admits a Mod(k) vertex magic labeling.

In this paper, we introduce trees with diameter less than four which satisfies the Mod(k) vertex magic labeling.

Preliminaries: In this section, we provide the basic notations related to this paper.

Classification:1: A tree of diameter one be the Path (P_2) with two vertices say v_1,v_2 and v_1v_2 be the edge of P_2. Let it be denoted by T_1.

Classification:2: A tree of diameter two be the Path (P_3) with three vertices say v_1,v_2, v_3 and v_1v_2, $v_2v_3$be the edges of P_3. Let it be denoted by T_2.

Classification: 3: The tree of diameter three of following types.

Type 1: Path(P_4) with four vertices say v_1,v_2, v_3, v_4 and the edge set is $\{v_iv_{i+1} : 1\leq i \leq 3\}$ of P_4. Let it be denoted by $T_3^{(1)}$.

Type 2: Path(P_4) with four vertices say v_1,v_2, v_3, v_4 and the edge set is $\{v_iv_{i+1} : 1\leq i \leq 3\}$ of P_4 and one pendant edge attached with v_2 or v_3. Let it be denoted by $T_3^{(2)}$.

Type 3: Path(P_4) with four vertices say v_1,v_2, v_3, v_4 and the edge set is $\{v_iv_{i+1} : 1\leq i \leq 3\}$of P_4. Let ($m\geq 1$) be the number of pendant edges attached on v_2 and ($n\geq 1$) be the pendant edges attached on v_3 if m - n={-1,0,1}. Let it be denoted by $T_3^{(m,n)}$.

Type 4: Bistar ($B_{m,n}$) if m - n={-1,0,1} for all m,n ≥ 1.

Remarks:
2.1. B(1,1) is isomorphic to $T_3^{(1)}$.
2.2. B(1,2) or B(2,1) is isomorphic to $T_3^{(2)}$.
2.3. B(m+1, n+1) is isomorphic to $T_3^{(m,n)}$ if m-n={-1,0,1} for all m,n ≥ 1.

Main Results: In this section, we prove the existence of Mod(k) vertex magic labeling of trees with diameter less than four.

Proposition 3.1: T_1 admits Mod(k) vertex magic labeling.
Proof: Let T_1 be the Path (P_2) of two vertices say v_1,v_2 and v_1v_2 be the edge of P_2.
Case(i): When k is odd.
Let w∈ V(T_1).
Define f: V(T_1)→$\{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right]+l+1, \left[\frac{k}{2}\right]+k\}$ by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w = v_1, \text{for } 0 \leq l \leq k-1,\\ \left[\frac{k}{2}\right] + l + 1, \text{if } w = v_2 \text{ for } 0 \leq l \leq k-2,\\ \left[\frac{k}{2}\right] + k, \text{if } w = v_2 \text{ for } l = k-1.\end{cases}$$

Clearly the mapping f is an injective. Hence for every edge (e=uv)∈E(T_1)

$$f(u)+f(v)=\begin{cases}k + l, \text{if } u = v_1, v = v_2 \text{ for } 0 \leq l \leq k-2,\\ k + k - 1, \text{if } u = v_1, v = v_2 \text{ for } l = k-1.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=l.
By the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping. Thus f admits a Mod(k) vertex magic labeling for T_1.
T_1 is a Mod(k)vertex magic graph if k is odd.

Case(ii): When k is even.
Let w∈ V(T_1).
Define f: V(T_1)→$\{\left[\frac{k}{2}\right], \left[\frac{k}{2}\right]+l, \left[\frac{k}{2}\right]+k\}$ by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w=v_1, \text{for } 0\le l\le k-1,\\ \left[\frac{k}{2}\right]+k, \text{if } w=v_2 \text{ for } l=0,\\ \left[\frac{k}{2}\right]+l, \text{if } w=v_2 \text{ for } 1\le l\le k-1.\end{cases}$$

Obviously f is an injective mapping. Hence for every edge (e=uv)∈E(T_1)

$$f(u)+f(v)=\begin{cases}2k, \text{if } u=v_1, v=v_2 \text{ for } l=0,\\ k+l, \text{if } u=v_1, v=v_2 \text{ for } 1\le l\le k-1.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=*l*.

According to the definition of Mod(k)

vertex magic labeling, the mapping f* is a constant mapping. Under the function f, there exists Mod(k) vertex magic labeling for T_1.

T_1 is a Mod(k)vertex magic graph if k is even.

Hence T_1 admits Mod(k) vertex magic labeling.

Proposition 3.2: T_2 admits Mod(k) vertex magic labeling.

Proof: Let T_2 be the Path (P_3) of three vertices say v_1,v_2,v_3 and $\{v_1v_2, v_2v_3\}$ be the edge set of P_3.

Case(i): When k is odd.

Let w∈ V(T_2).

Define f:V(T_2)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+*l*+1, $\left[\frac{k}{2}\right]$+k, $\left[\frac{k}{2}\right]$+k+*l* +1, $\left[\frac{k}{2}\right]$+2k}by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right]+\frac{k}{2}(i-1), \text{if } w=v_i, \text{for } 0\le l\le k-2, i=1,3,\\ \left[\frac{k}{2}\right]+l+1, \text{if } w=v_2 \text{ for } 0\le l\le k-2,\\ \left[\frac{k}{2}\right]+k(i-1), \text{if } w=v_i \text{ for } l=k-1, 1\le i\le 3.\end{cases}$$

Clearly the mapping f is an injective. Hence for every edge (e=uv)∈E(T_2)

$$f(u)+f(v)=\begin{cases}k+l, \text{if } u=v_1, v=v_2 \text{ for } 0\le l\le k-2,\\ 2k+l, \text{if } u=v_2, v=v_3 \text{ for } 0\le l\le k-2,\\ k(2i-1)+k-1, \text{if } u=v_i, v=v_{i+1} \text{ for } l=k-1, 1\le i\le 2.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=*l*.

By the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping. Thus f admits a Mod(k) vertex magic labeling for T_2.

T_2 is a Mod(k)vertex magic graph if k is odd .

Case(ii): When k is even.

Let w∈ V(T_2).

Define f: V(T_2)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+ *l*, $\left[\frac{k}{2}\right]$+k, $\left[\frac{k}{2}\right]$+k+ *l* ,
$\left[\frac{k}{2}\right]$+2k}by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right]+k(i-1), \text{if } w=v_i \text{ for } l=0, 1\le i\le 3,\\ \left[\frac{k}{2}\right]+\frac{k}{2}(i-1), \text{if } w=v_i, \text{for } 1\le l\le k-1, i=1,3,\\ \left[\frac{k}{2}\right]+l, \text{if } w=v_2 \text{ for } 1\le l\le k-1.\end{cases}$$

Clearly the mapping f is an injective. Hence for every edge (e=uv)∈E(T_2)

$$f(u)+f(v)=\begin{cases}k(2i), \text{if } u=v_i, v=v_{i+1} \text{ for } l=0, 1\le i\le 2,\\ k+l, \text{if } u=v_1, v=v_2 \text{ for } 1\le l\le k-1,\\ 2k+l, \text{if } u=v_2, v=v_3 \text{ for } 1\le l\le k-1.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=*l*.

By the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping.

Under the function f, there exists Mod(k) vertex magic labeling for T_2.

T_2 is a Mod(k)vertex magic graph if k is even. Hence T_2 admits Mod(k) vertex magic labeling.

Theorem:3.3: Bistar graph (Bm,n) admits Mod(k) vertex magic labeling if m-n={-1,0,1} for all m,n≥1.
Proof: Let G=(V(G),E(G)) be a Bistar graph ($B_{m,n}$) having (m+n+2) vertices and (mn+1)edges.
Let V(G)={u}∪{v}∪u_i: $1 \le i \le m$ } ∪ {v_j: $1 \le j \le n$} and E(G)={uv}∪ {uu_i: $1 \le i \le m$}∪{vv_j: $1 \le j \le n$ }.
Case (a):- When m - n = -1.
Subcase(i): k is odd.
Let w∈ V(G).
Define f: V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]+l+1,\dots,\left[\frac{k}{2}\right]+k(2n)$} by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w=u, \text{for } 0 \le l \le k-1,\\ \left[\frac{k}{2}\right]+l+1, \text{if } w=v \text{ for } 0 \le l \le k-2,\\ \left[\frac{k}{2}\right]+k, \text{if } w=v \text{ for } l=k-1,\\ \left[\frac{k}{2}\right]+\frac{k}{2}(2i)+l+1, \text{if } w=u_i, \text{for } 0 \le l \le k-2, 1 \le i \le n-1,\\ \left[\frac{k}{2}\right]+k(i+1), \text{if } w=u_i \text{ for } l=k-1, 1 \le i \le n-1,\\ \left[\frac{k}{2}\right]+\frac{k}{2}(2j), \text{if } w=v_j, \text{for } 0 \le l \le k-2, 1 \le j \le n,\\ \left[\frac{k}{2}\right]+k(m+j+1), \text{if } w=v_j \text{ for } l=k-1, 1 \le j \le n.\end{cases}$$

Clearly the mapping f is an injective. Hence for every edge (e=uv)∈E(G)

$$f(u)+f(v)=\begin{cases}k+l, \text{if } u=u, v=v \text{ for } 0 \le l \le k-1,\\ k(i+1)+l, \text{if } u=u, v=u_i \text{ for } 0 \le l \le k-2, 1 \le i \le n-1,\\ k(i+1)+k-1, \text{if } u=u, v=u_i \text{ for } l=k-1, 1 \le i \le n-1,\\ k(j+1)+l, \text{if } u=v, v=v_j \text{ or } 0 \le l \le k-2, 1 \le j \le n,\\ k(m+j+2)+k-1, \text{if } u=v, v=v_j \text{ for } l=k-1, 1 \le j \le n.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=*l*.
According to the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping.
Thus f admits a Mod(k) vertex magic labeling for G. $B_{m,n}$ admits Mod(k) vertex magic labeling if k is odd, m-n=-1 for all m,n≥1.
Subcase(ii): k is even.
Let w∈ V(G).
Define f: V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]+l,\dots,\left[\frac{k}{2}\right]+k(2n)$} by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w=u, \text{for } 0 \le l \le k-1,\\ \left[\frac{k}{2}\right]+k, \text{if } w=v \text{ for } l=0,\\ \left[\frac{k}{2}\right]+l+1, \text{if } w=v \text{ for } 1 \le l \le k-1,\\ \left[\frac{k}{2}\right]+k(i+1), \text{if } w=u_i \text{ for } l=0, 1 \le i \le n-1,\\ \left[\frac{k}{2}\right]+k(m+j+1), \text{if } w=v_j \text{ for } l=0, 1 \le j \le n,\\ \left[\frac{k}{2}\right]+\frac{k}{2}(2j), \text{if } w=v_j, \text{for } 1 \le l \le k-1, 1 \le j \le n.\end{cases}$$

Obviously the mapping f is an injective. Hence for every edge (e=uv)∈E(G)

$$f(u)+f(v)=\begin{cases}k+l, \text{if } u=u, v=v \text{ for } 0 \le l \le k-1,\\ k(i+2), \text{if } u=u, v=u_i \text{ for } l=0, 1 \le i \le n-1,\\ k(i+1)+l, \text{if } u=u, v=u_i \text{ for } 1 \le l \le k-1, 1 \le i \le n-1,\\ k(m+j+3), \text{if } u=v, v=v_j \text{ for } l=0, 1 \le j \le n,\\ k(j+1)+l, \text{if } u=v, v=v_j \text{ or } 1 \le l \le k-1, 1 \le j \le n.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=*l*.
By the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping.

Thus f admits a Mod(k) vertex magic labeling for G. $B_{m,n}$ admits Mod(k) vertex magic labeling if k is even, m-n = -1 for all m,n≥1.

Case (b):- When m-n=0.
Subcase(iii): k is odd.
Let w∈ V(G).
Define f: V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+ l+1,… , $\left[\frac{k}{2}\right]$+k(2n+1)} by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w = u, \text{for } 0 \le l \le k-1,\\ \left[\frac{k}{2}\right] + l + 1, \text{if } w = v \text{ for } 0 \le l \le k-2,\\ \left[\frac{k}{2}\right] + k, \text{if } w = v \text{ for } l = k-1,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2i) + l + 1, \text{if } w = u_i, \text{for } 0 \le l \le k-2, 1 \le i \le n,\\ \left[\frac{k}{2}\right] + k(i+1), \text{if } w = u_i \text{ for } l = k-1, 1 \le i \le n,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2j), \text{if } w = v_j, \text{for } 0 \le l \le k-2, 1 \le j \le n,\\ \left[\frac{k}{2}\right] + k(m+j+1), \text{if } w = v_j \text{ for } l = k-1, 1 \le j \le n.\end{cases}$$

It is easy to see that the mapping f is an injective. Hence for every edge (e=uv)∈E(G)

$$f(u)+f(v)=\begin{cases}k + l, \text{if } u = u, v = v \text{ for } 0 \le l \le k-1,\\ k(i+1) + l, \text{if } u = u, v = u_i \text{ for } 0 \le l \le k-2, 1 \le i \le n,\\ k(i+1) + k - 1, \text{if } u = u, v = u_i \text{ for } l = k-1, 1 \le i \le n,\\ k(j+1) + l, \text{if } u = v, v = v_j \text{ or } 0 \le l \le k-2, 1 \le j \le n,\\ k(m+j+2) + k - 1, \text{if } u = v, v = v_j \text{ for } l = k-1, 1 \le j \le n.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=l.
According to the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping.
Thus f admits Mod(k) vertex magic labeling for G.
$B_{m,n}$ is a Mod(k) vertex magic graph if k is odd, m-n=0 for all m,n≥1.
Sub Case(iv): k is even.
Let w∈ V(G).
Define f: V(G)→{$\left[\frac{k}{2}\right]$,$\left[\frac{k}{2}\right]$+l,… , $\left[\frac{k}{2}\right]$+k(2n+1)} by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w = u, \text{for } 0 \le l \le k-1,\\ \left[\frac{k}{2}\right] + k, \text{if } w = v \text{ for } l = 0,\\ \left[\frac{k}{2}\right] + l, \text{if } w = v \text{ for } 1 \le l \le k-1,\\ \left[\frac{k}{2}\right] + k(i+1), \text{if } w = u_i \text{ for } l = 0, 1 \le i \le n,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2i) + l, \text{if } w = u_i, \text{for } 1 \le l \le k-1, 1 \le i \le n,\\ \left[\frac{k}{2}\right] + k(m+j+1), \text{if } w = v_j \text{ for } l = 0, 1 \le j \le n,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2j), \text{if } w = v_j, \text{for } 1 \le l \le k-1, 1 \le j \le n.\end{cases}$$

Clearly f is an injective mapping. Hence for every edge (e=uv)∈E(G)

$$f(u)+f(v)=\begin{cases}k + l, \text{if } u = u, v = v \text{ for } 0 \le l \le k-1,\\ k(i+2), \text{if } u = u, v = u_i \text{ for } l = 0, 1 \le i \le n,\\ k(i+1) + l, \text{if } u = u, v = u_i \text{ for } 1 \le l \le k-1, 1 \le i \le n,\\ k(m+j+3), \text{if } u = v, v = v_j \text{ for } l = 0, 1 \le j \le n,\\ k(j+1) + l, \text{if } u = v, v = v_j \text{ or } 1 \le l \le k-1, 1 \le j \le n.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=l.
According to the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping. Thus f admits a Mod(k) vertex magic labeling for G.
$B_{m,n}$ is a Mod(k) vertex magic graph if k is even, m-n=0 for all m,n≥1.

Case(c):- When m-n=1.
Sub Case(v): k is odd.
Let w∈ V(G).
Define f: V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+ l+1, … , $\left[\frac{k}{2}\right]$+k(2n+2)} by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w = u, \text{for } 0 \le l \le k-1,\\ \left[\frac{k}{2}\right] + l + 1, \text{if } w = v \text{ for } 0 \le l \le k-2,\\ \left[\frac{k}{2}\right] + k, \text{if } w = v \text{ for } l = k-1,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2i), \text{if } w = u_i, \text{for } 0 \le l \le k-2, 1 \le i \le n+1,\\ \left[\frac{k}{2}\right] + k(m+i+1), \text{if } w = u_i \text{ for } l = k-1, 1 \le i \le n+1,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2j) + l + 1, \text{if } w = v_j, \text{for } 0 \le l \le k-2, 1 \le j \le n,\\ \left[\frac{k}{2}\right] + k(j+1), \text{if } w = v_j \text{ for } l = k-1, 1 \le j \le n.\end{cases}$$

Obviously f is an injective mapping. Hence for every edge (e=uv)∈E(G)

$$f(u)+f(v)=\begin{cases}k + l, \text{if } u = u, v = v \text{ for } 0 \le l \le k-1,\\ k(i+1) + l, \text{if } u = u, v = u_i \text{ for } 0 \le l \le k-2, 1 \le i \le n+1,\\ k(m+i+2) + k - 1, \text{if } u = u, v = u_i \text{ for } l = k-1, 1 \le i \le n+1,\\ k(j+1) + l, \text{if } u = v, v = v_j \text{ or } 0 \le l \le k-2, 1 \le j \le n,\\ k(j+1) + k - 1, \text{if } u = v, v = v_j \text{ for } l = k-1, 1 \le j \le n.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=l.
According to the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping. Thus f admits a Mod(k) vertex magic labeling for G.
$B_{m,n}$ admits Mod(k) vertex magic labeling if k is odd, m-n=1 for all m,n≥1.
Sub Case(vi): k is even.
Let w∈ V(G).
Define f: V(G)→{$\left[\frac{k}{2}\right]$, $\left[\frac{k}{2}\right]$+ l,… , $\left[\frac{k}{2}\right]$+k(2n+2)} by

$$f(w)=\begin{cases}\left[\frac{k}{2}\right], \text{if } w = u, \text{for } 0 \le l \le k-1,\\ \left[\frac{k}{2}\right] + k, \text{if } w = v \text{ for } l = 0,\\ \left[\frac{k}{2}\right] + l, \text{if } w = v \text{ for } 1 \le l \le k-1,\\ \left[\frac{k}{2}\right] + k(m+i+1), \text{if } w = u_i \text{ for } l = 0, 1 \le i \le n+1,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2i), \text{if } w = u_i, \text{for } 1 \le l \le k-1, 1 \le i \le n+1,\\ \left[\frac{k}{2}\right] + k(j+1), \text{if } w = v_j \text{ for } l = 0, 1 \le j \le n,\\ \left[\frac{k}{2}\right] + \frac{k}{2}(2j) + l, \text{if } w = v_j, \text{for } 1 \le l \le k-1, 1 \le j \le n.\end{cases}$$

Obviously the mapping f is an injective. Hence for every edge (e=uv)∈E(G)

$$f(u)+f(v)=\begin{cases}k + l, \text{if } u = u, v = v \text{ for } 0 \le l \le k-1,\\ k(m+i+3), \text{if } u = u, v = u_i \text{ for } l = 0, 1 \le i \le n+1,\\ k(i+1) + l, \text{if } u = u, v = u_i \text{ for } 1 \le l \le k-1, 1 \le i \le n+1,\\ k(j+2), \text{if } u = v, v = v_j \text{ for } l = 0, 1 \le j \le n,\\ k(j+1) + l, \text{if } u = v, v = v_j \text{ or } 1 \le l \le k-1, 1 \le j \le n.\end{cases}$$

Hence f*(e =uv) = (f(u)+f(v)) (mod k)=l.
According to the definition of Mod(k) vertex magic labeling, the mapping f* is a constant mapping.
Thus f admits a Mod(k) vertex magic labeling for G.
$B_{m,n}$ admits Mod(k) vertex magic labeling if k is even, m-n=1 for all m,n≥1.
Hence Bistar ($B_{m,n}$) admits Mod(k) vertex magic labeling if m-n={-1,0,1} for all m,n≥1.

Illustration 1: Bistar (B4,5) is a Mod(2) vertex magic graph for l =1 is shown in Figure 1.

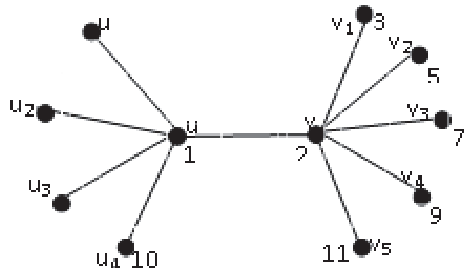

Figure 1

Corollary 3.4: $T_3^{(1)}$ admits Mod(k) vertex magic labeling.
Proof: By Theorem 3.3 and Remark 2.1, it follows that $T_3^{(1)}$ admits Mod(k) vertex magic labeling.

Corollary 3.5: $T_3^{(2)}$ admits Mod(k) vertex magic labeling.
Proof: By Theorem 3.3 and Remark 2.2, it follows that $T_3^{(2)}$ admits Mod(k) vertex magic labeling.

Corollary 3.6: If m-n={-1,0,1} then $T_3^{(m,n)}$ admits Mod(k) vertex magic labeling for all m,n≥1.
Proof: By Theorem 3.3 and Remark 2.3, it follows that $T_3^{(m,n)}$ admits Mod(k) vertex magic labeling if m-n={-1,0,1} for all m,n≥1.

Conclusion: In this paper we have discussed that some of the trees with diameter less than four are Mod(k) vertex magic graphs. Analogues work can be carried by us for other families also.

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